

Critical Speed Yaw: Special Situations

Ofc. Wade Bartlett, PE
Mechanical Forensics
Engineering Services, LLC

1

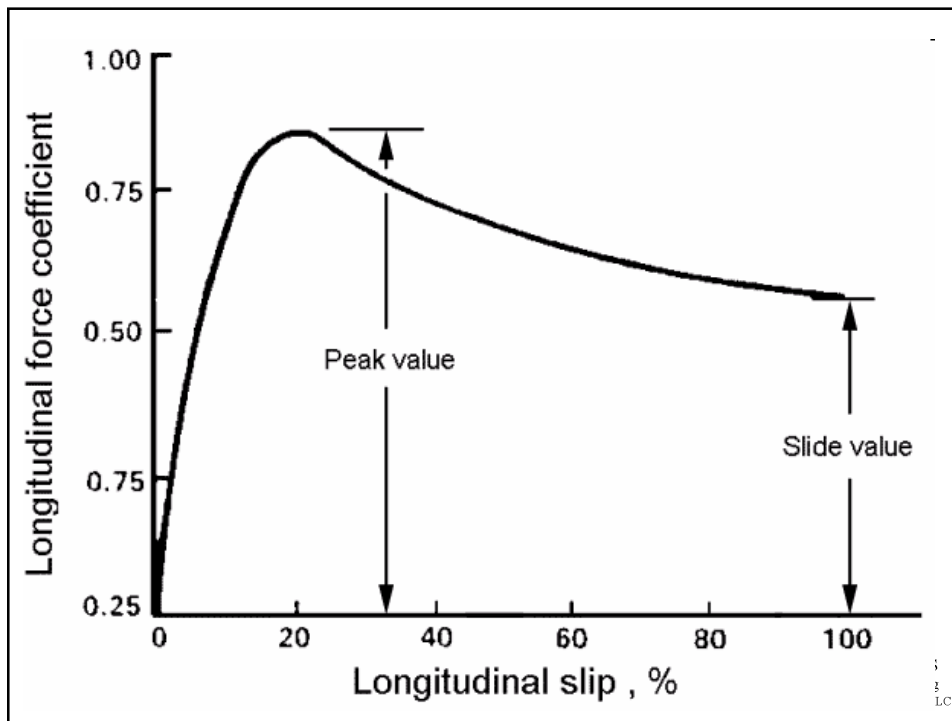


Outline

- Friction: -Longitudinal Peak/Avg
-Lateral (Cornering)
- Friction Circle & Combined Forces
- CSY Theory (and built-in assumptions)
- CSY in the Literature
- Nuggets of “***Common Wisdom***”
 - Good, Bad, Ugly
- Special Cases:
 - Grass, Gravel, Split-Co, ESC

Friction

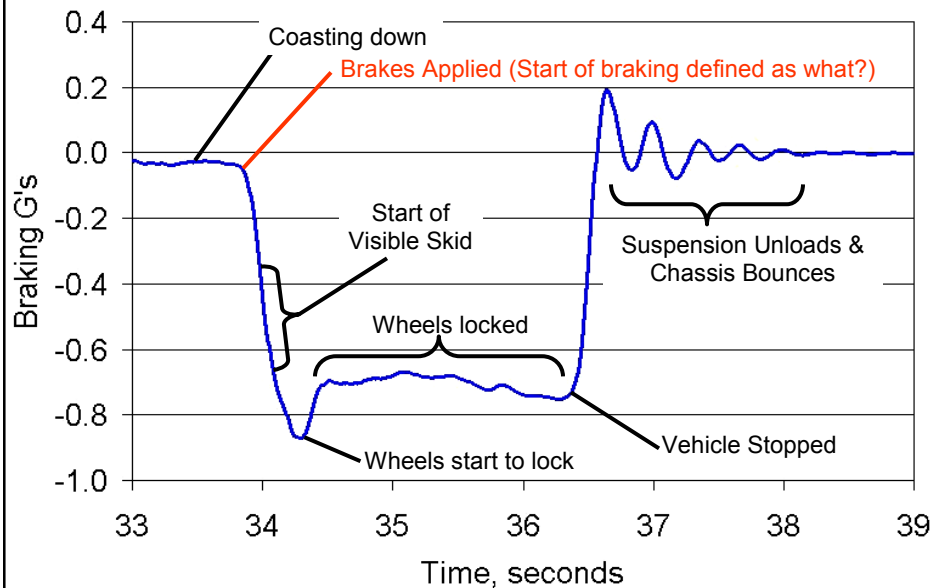
- Forces between tire & roadway develop (primarily) as a result of slip between the two surfaces.
- Longitudinal force (Brake/Accel) comes from the wheel not rolling at same speed as vehicle traveling in direction it's facing
- Slip Ratio of 0% = free rolling wheel
- Slip Ratio 100% = locked wheel
- Peak Longitudinal Friction @ SR ~15-25%



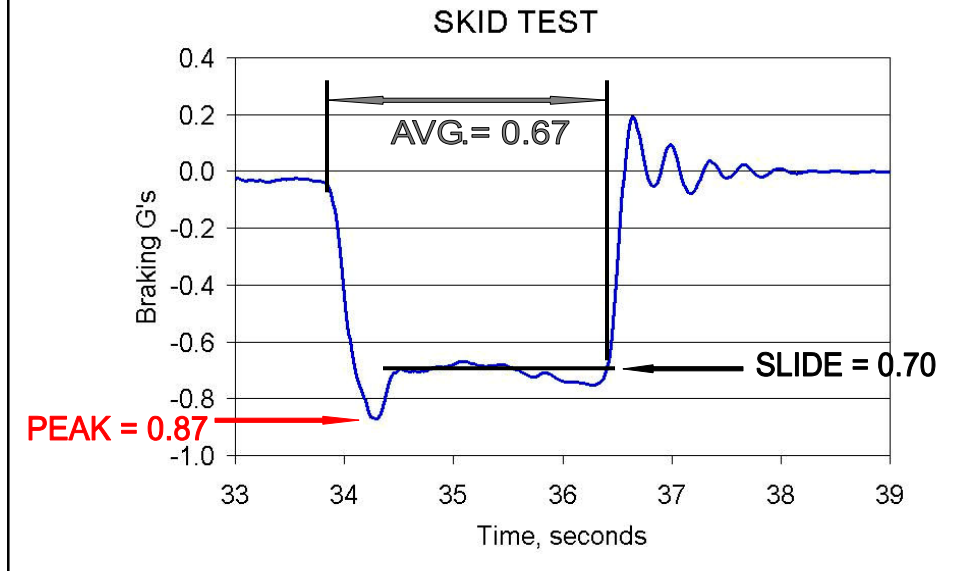
Longitudinal Friction

- Are all friction measurements the same?
- Is the f determined with a shot marker the same as the f from measuring test skids?
- **No.** But is that ok?
- **Yes.** Just need to know which one we're using
- Average (shotmarker ~ brake timer)
- Peak / Slide (or Plateau)

Typical Skid Test - Accelerometer



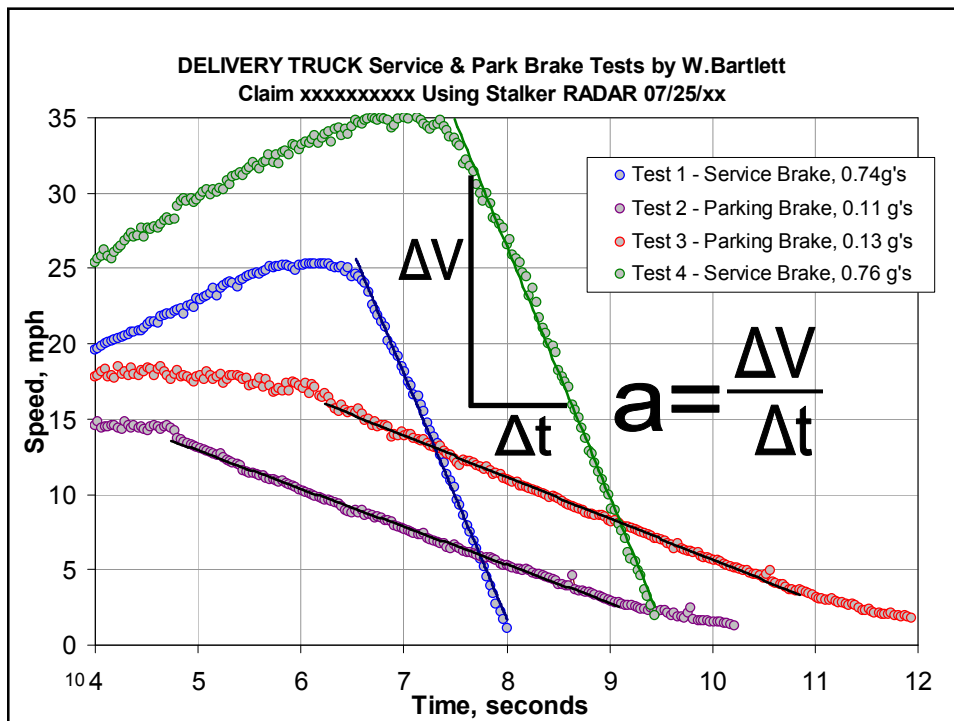
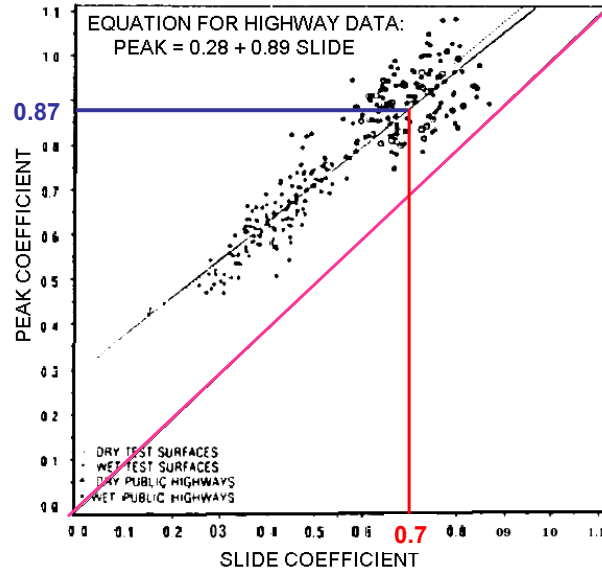
Which Friction Value?



FRICITION SAE 890638

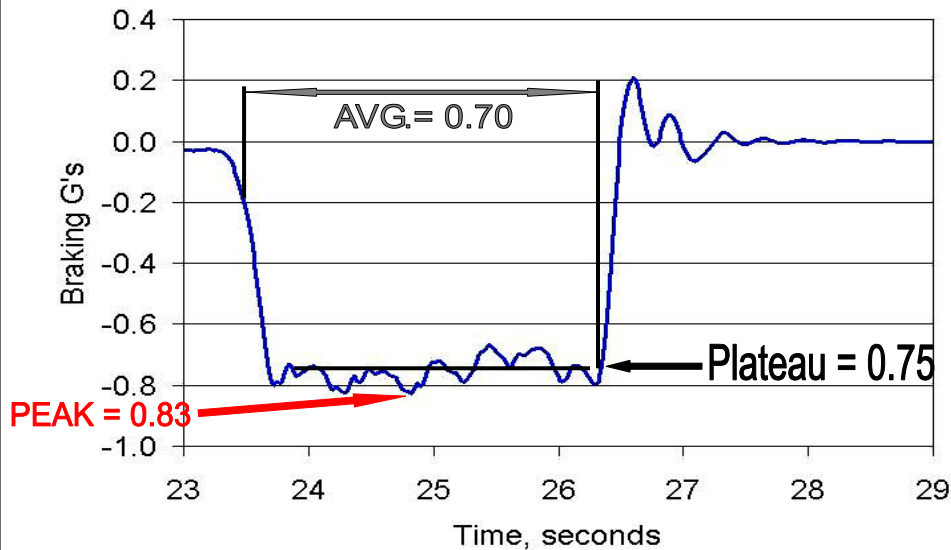
- “SAE tire braking traction survey—A comparison of public highways and test surfaces”
- 2 car makers & 6 tire companies provided tire braking traction data from their respective test surfaces and several public highways. Peak and slide coefficients were measured on wet and dry surfaces at two speeds and two loads.
- Showed f_{PEAK} always higher than f_{SLIDING}
- Related to STATIC vs. KINETIC friction

FRICITION SAE 890638



Which Friction Value?

SKID TEST - ABS

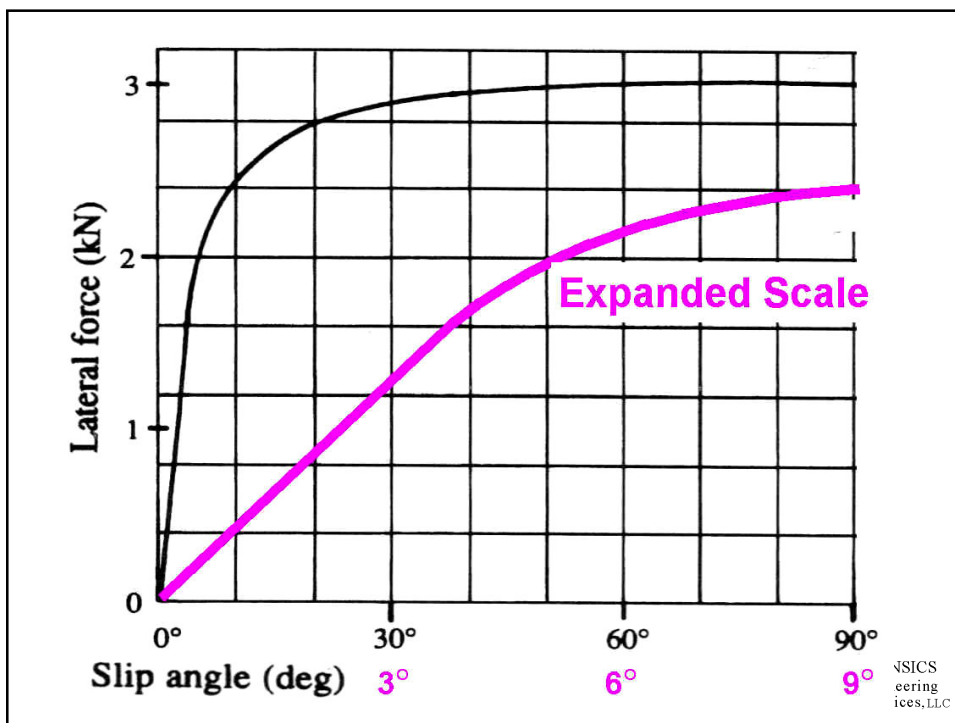
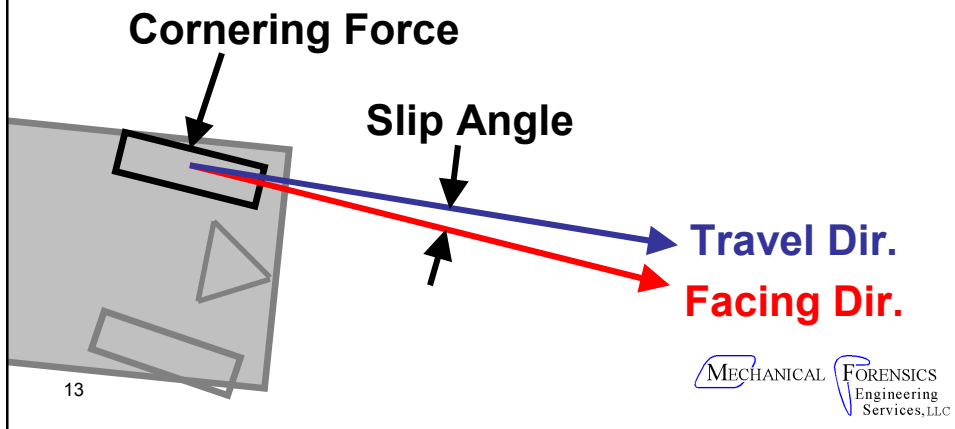


Brake Balance

- MY2000 and newer cars required by law to lock FRONT wheels first under all friction and loading conditions. (FMVSS 135)
- All four brakes NEVER working at peak value at the same time.
- Peak value measured during skid test will always be a little lower than the actual peak for the tires/roadway.

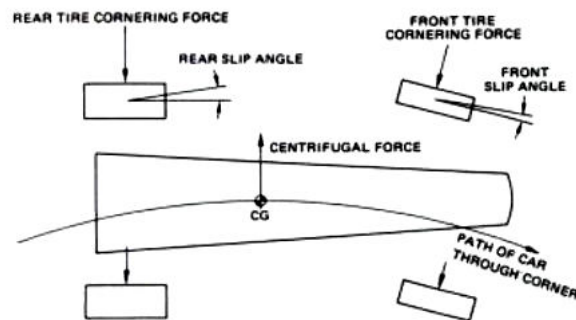
Lateral Friction

- Develops as a function of slip angle between the tire and the roadway



Lateral Friction

- Front/Rear tires not all pointed same direction, but they are close! Treat car as one unit, averaging the four tire forces.



15

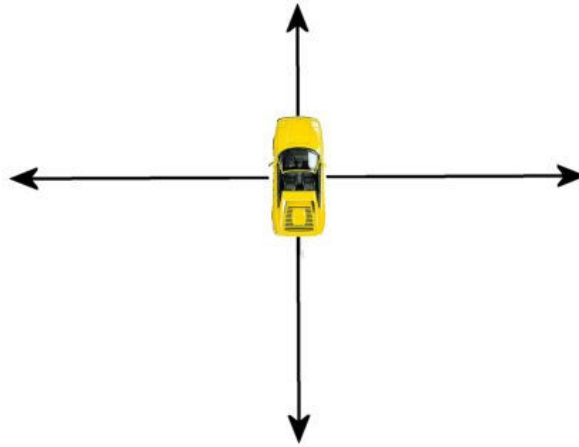
MECHANICAL FORENSICS
Engineering
Services, LLC

Lateral Friction

- Hard to measure at a scene
- Lateral Peak value generally same or slightly higher than Long. Peak for a tire.
- 4 tires near peak lateral friction simultaneously during CSY.
- Lateral peak > Long.Peak from Skid test due to imperfect brake balance.
- Measured Long.Peak will be a slightly conservative approximation of Lateral Peak

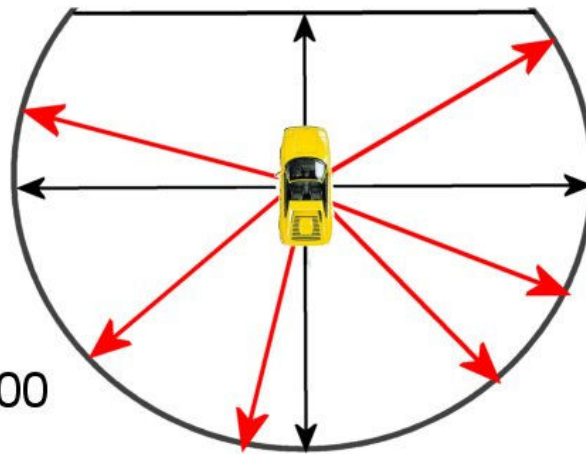
Friction - On Axis

$$f = 1.00$$



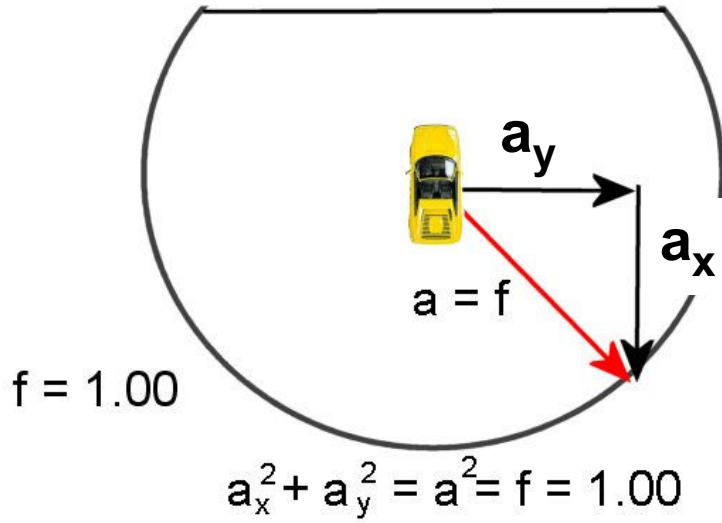
Friction Circle - Off Axis

$$f = 1.00$$

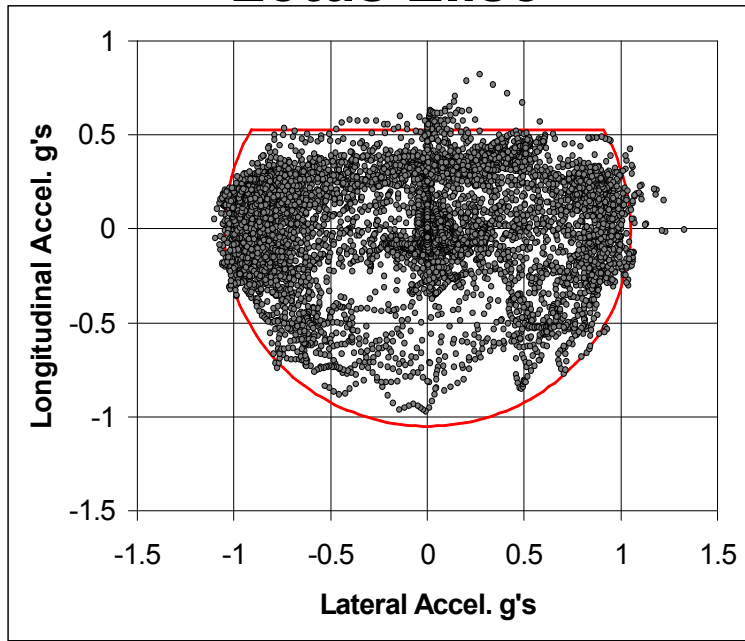


$$r \text{ everywhere} = f = a = 1.00$$

Friction Circle

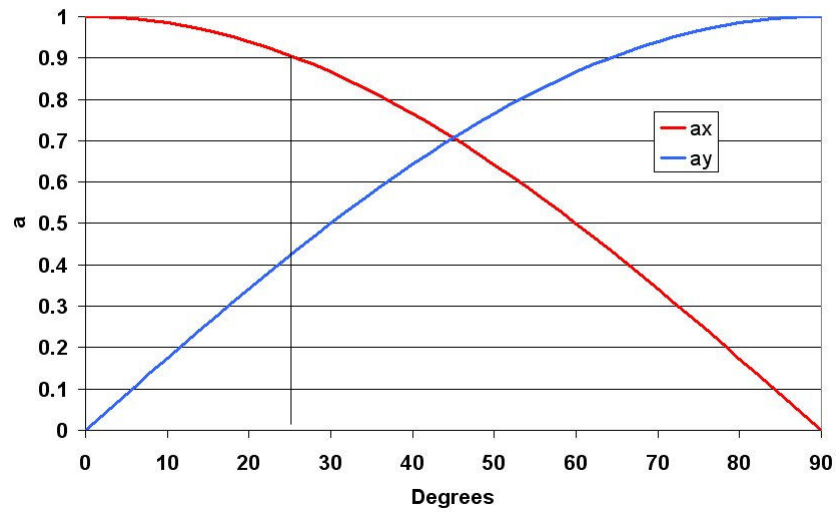


Lotus Elise



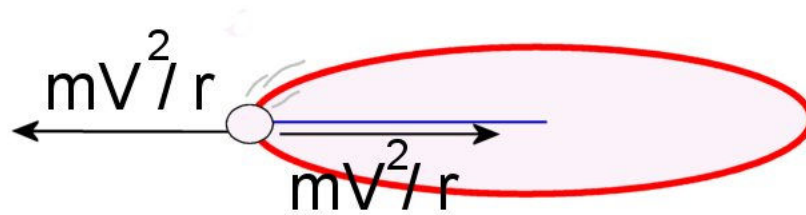
Friction Circle: Simple Model

Ax vs Ay

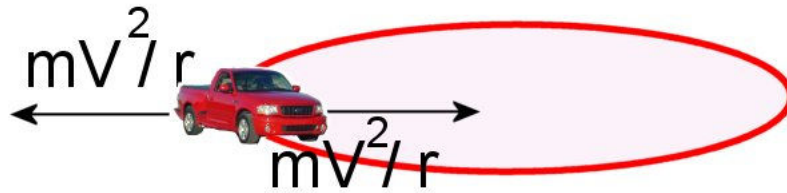


CSY: The Theory

- A ball on a string



The CSY Theory: A Ball on a String



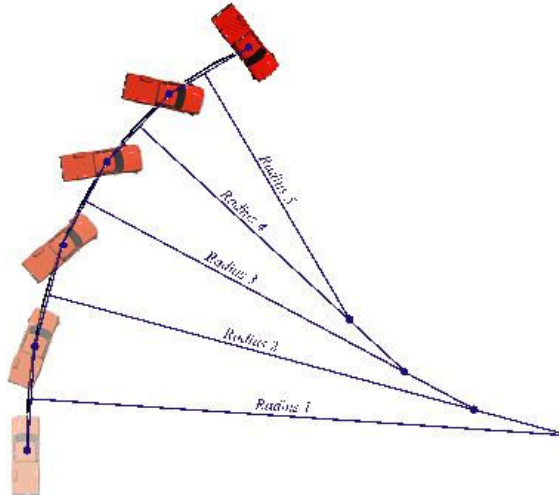
The CSY Theory: A Truck on a String



$$S = 3.86\sqrt{r f}$$

Assumptions

1. Vehicle follows a circular path



Assumptions

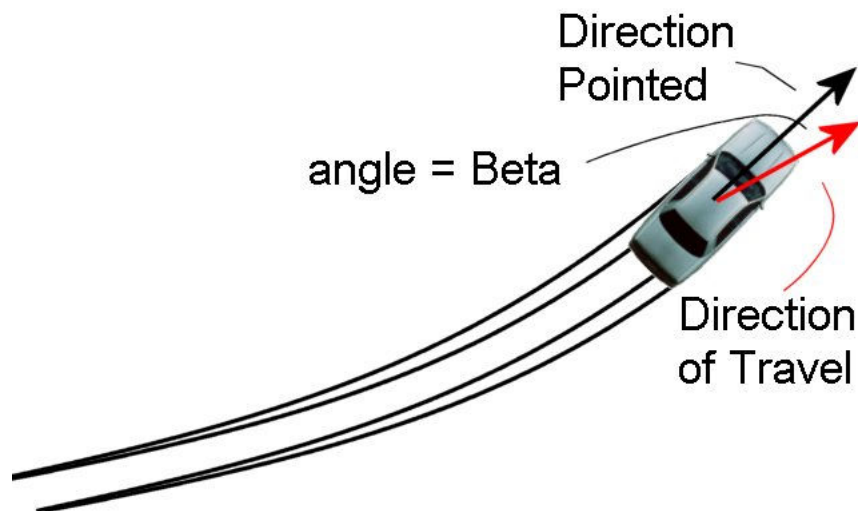
1. Vehicle follows a circular path.

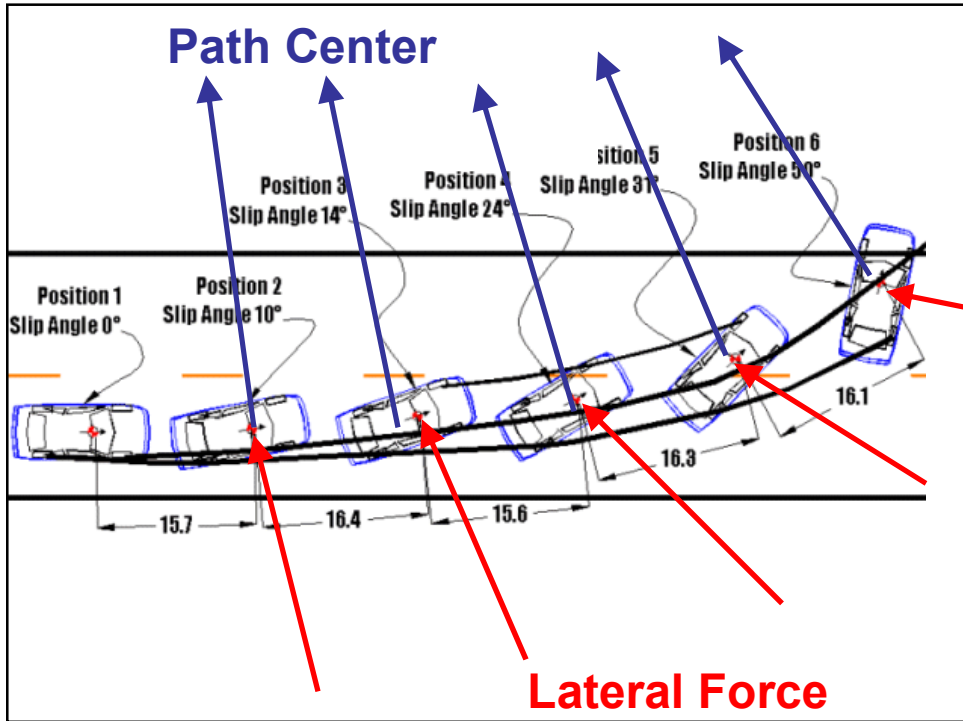
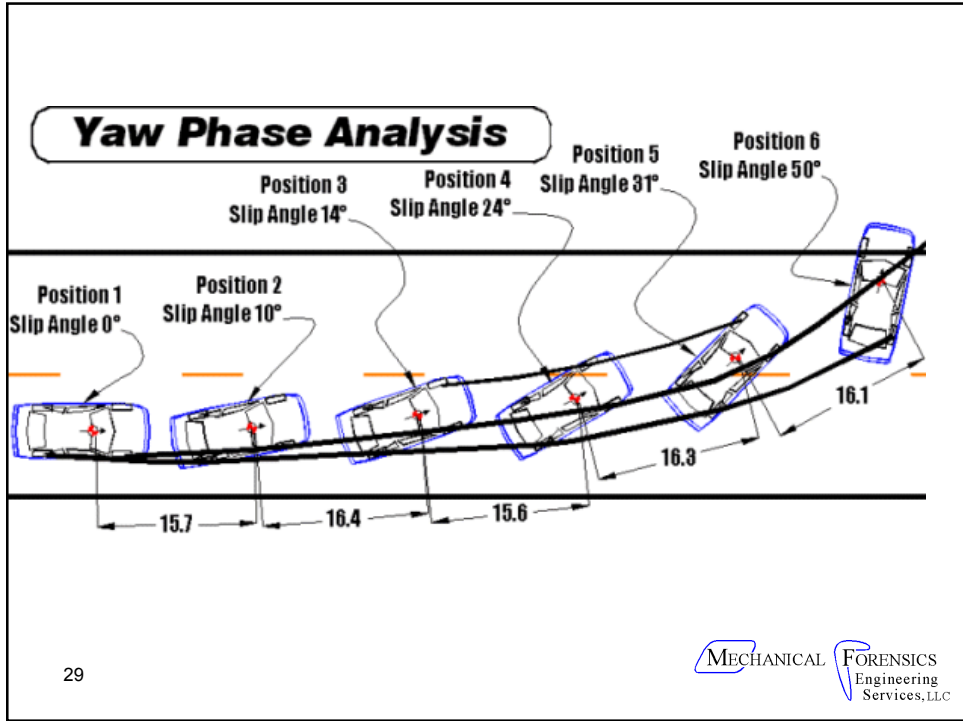
- Masory et al 2005-01-1189
- “Validation of the Circular Trajectory Assumption in Critical Speed”
- Circularity is a pretty good assumption over short distances.

Assumptions

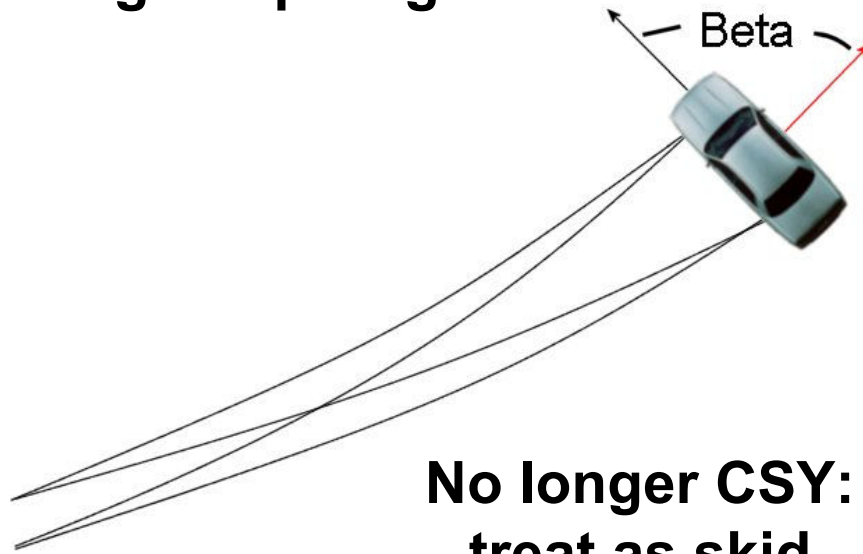
1. Vehicle follows a circular path.
2. Slip angle low enough that lateral friction still pointed pretty much along path radius.

Low Slip Angle

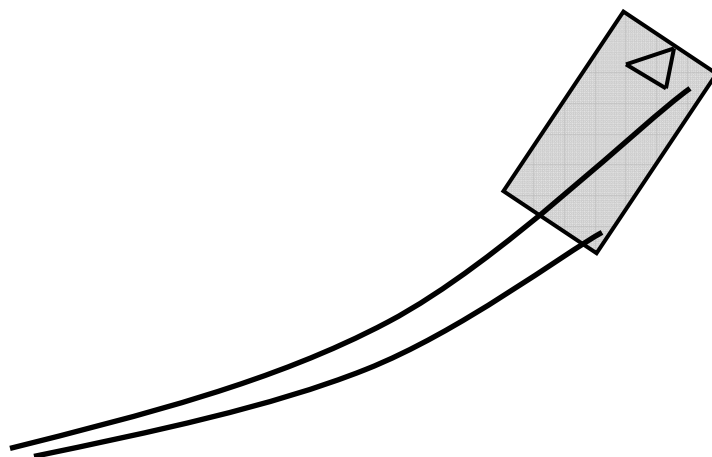




High Slip Angle

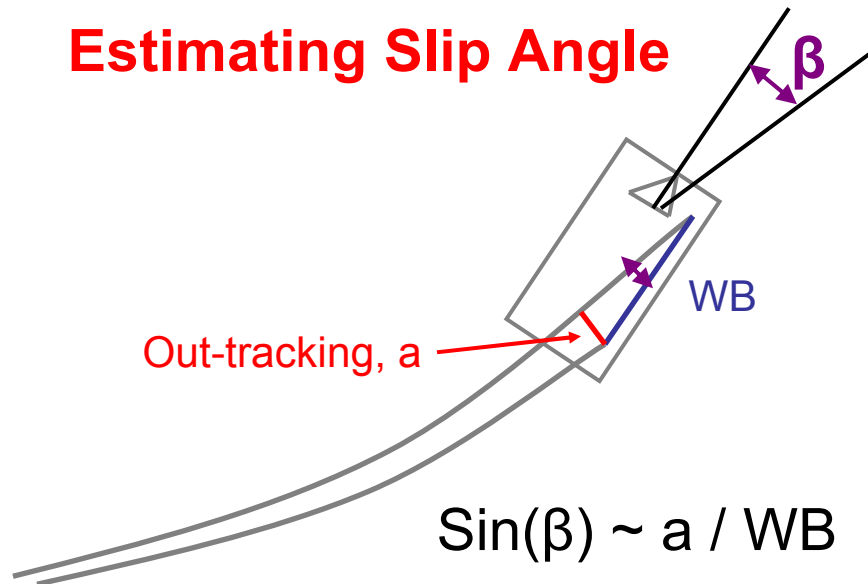


Estimating Slip Angle



32

Estimating Slip Angle



33

MECHANICAL FORENSICS
Engineering
Services, LLC

Estimating Slip Angle

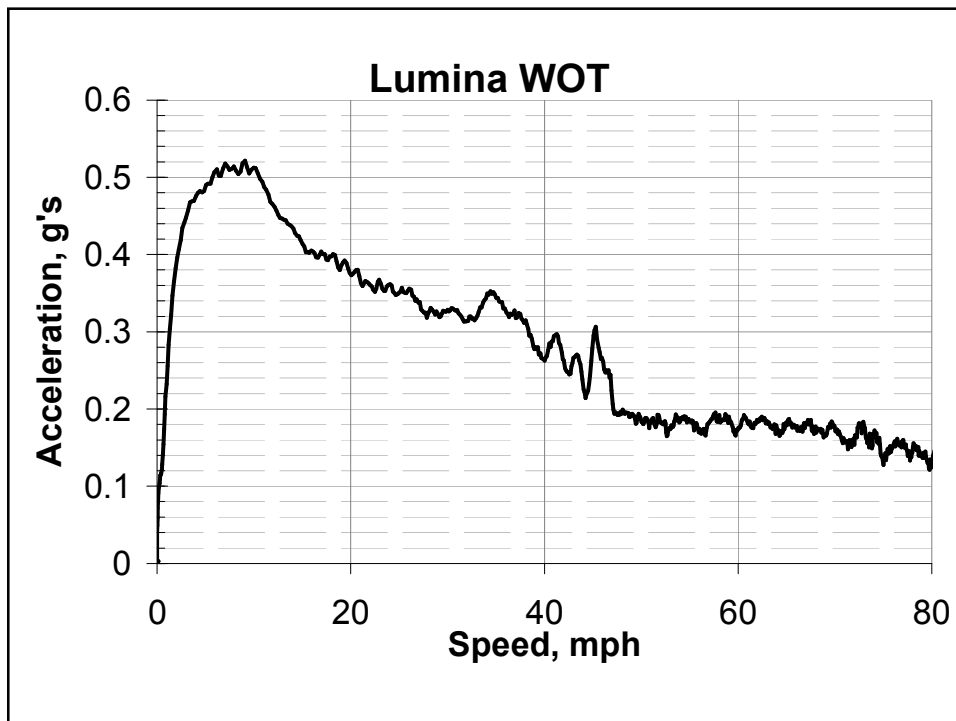
- $\sin(\beta) = \text{outtracking} / WB$
- $WB * \sin(\beta) = \text{outtracking}$
- $\sin(25 \text{ degrees}) = 0.423$
- Slip angle of 25 degrees when out-tracking about 42% of WB
- If $WB=9\text{ft}$, gap b/t marks=3.8ft, $\sim 1/2$ TW?

34

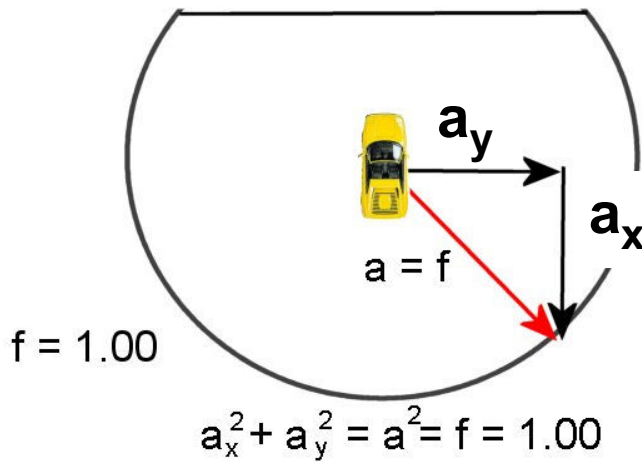
MECHANICAL FORENSICS
Engineering
Services, LLC

Assumptions

1. Vehicle follows a circular path.
2. Slip angle low enough that lateral friction still pointed pretty much along path radius.
3. **Longitudinal acceleration (braking or acceleration) is insufficient to degrade lateral friction too much.**



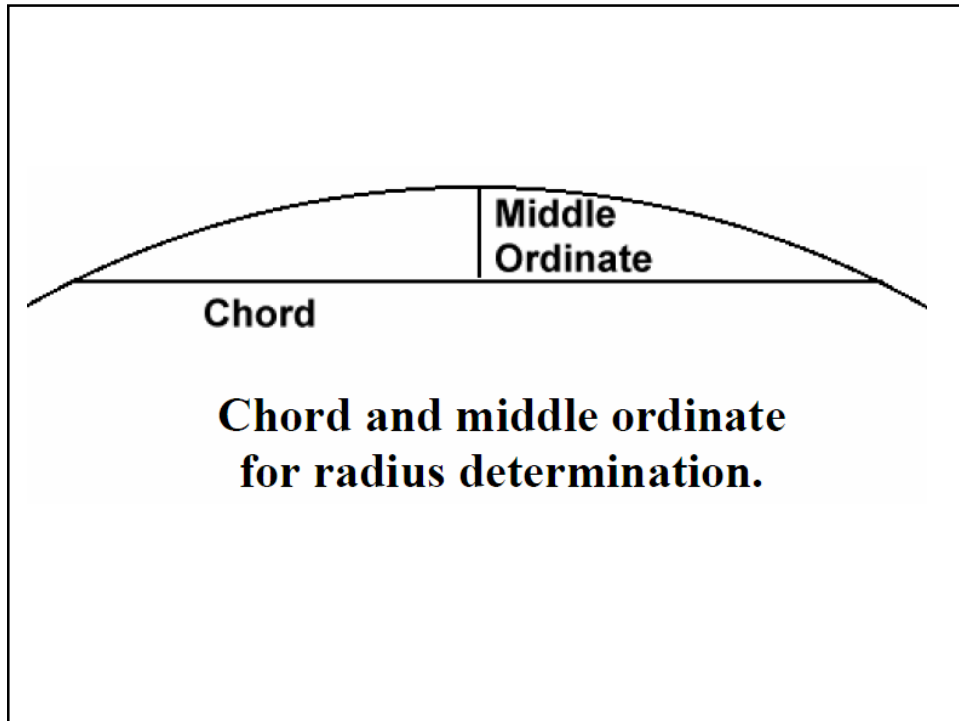
Modest a_x doesn't significantly affect Lateral: CSY tolerant of braking/accel.



Assumptions

1. Vehicle follows a circular path
2. Slip angle low enough that lateral friction still pointed pretty much along path radius.
3. Longitudinal acceleration insufficient to degrade lateral friction too much.
4. **Vehicle near ~peak lateral traction**
5. **Vehicles are essentially the same**
6. **Radius can be measured accurately**

38



$$R = \frac{C^2}{8M} + \frac{M}{2}$$

R = radius, meters or feet

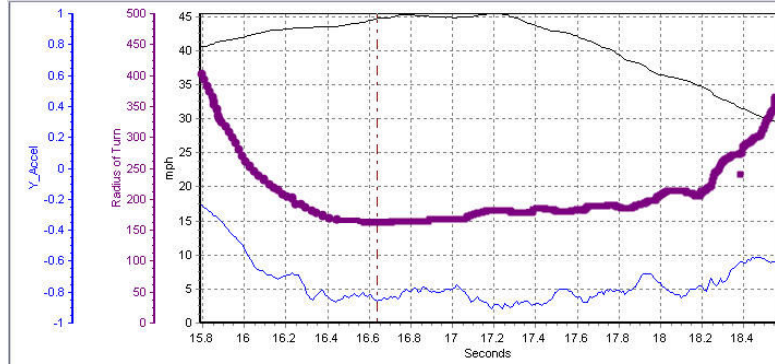
C = chord, meters or feet

M = middle ordinate, meters or feet

Radius Measurement

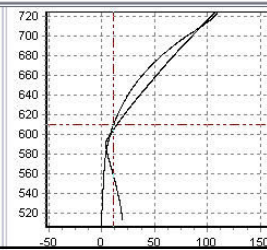
- Measuring radii was examined at WREX2000
- SD = 8 feet for a 188-ft radius yawmark (~1mph), but only 4-ft for chalked mark (see SAE 2002-01-0546, Bartlett et al)

Hand-Measured Radius = 164 feet



	20050918_43			
Run Time	0 Minute 52.6			
Cursor (Seconds)	16.63			
<input checked="" type="checkbox"/> Speed (mph)	44.65			
<input checked="" type="checkbox"/> Radius of Turn (feet)	166.94			
<input checked="" type="checkbox"/> V_Accel (g)	-0.859			

Radius = 166.9 ft



The Literature

Reveley 890635

- “A Comparison Study of Skid and Yaw Marks”
- Mostly documenting striations in the marks – some good descriptions and sketches
- No discussion of how the measuring was done
- Calc'd speeds as an aside using both peak and average mu (found with G-analyst).
- All calc'd speeds lower than speed measured at wheel during yaw
- Concluded "reasonably accurate method"

Lambourn 940723

- Follow on to 89 paper in J.For.Sci Soc.
- Used 15m chords
- Hand-held radar inside car observed and recorded manually at point of yaw initiation
- 4 vehicles, heavy ABS braking and coasting
- CONCLUSIONS
 - ABS braking generated less out-tracking
 - ABS cycling could not be ID'd in marks
 - CSF gave accuracy of $\pm 10\%$ of true
 - Heavy braking exacerbated under-estimation

Shelton, ARJ Jan/Feb 95, Vol.7(1)

- 94 tests / 15 years (79-94) CHP MAIT training
- Much of the raw data for all tests is provided
- Most tests were coasting, some braking/accel.
- Showed how calculated speed becomes very sensitive with small MO (Figure 4)
- CONCLUSIONS:
 - Chords less than 25 feet should be avoided
 - Chords up to 50 feet worked reasonably well
 - Friction values from skid testing worked well

Dickerson 950137

- “Evaluation of vehicle velocity predictions using CSF”
- Concrete airport apron, EscortGT w/ ballast/outriggers
- Accel. around 100ft circle to max speed (~3/4 turn)
- STEP, and DOUBLE STEP (2 tests each)
- Accel reported is 0.5 second moving average
- Double step marks > 200ft long, recovered from 27 degree sideslip angle
- Concluded:
 - High error at slip>25 deg. (up to 61 deg. reported)
 - Best results with high lateral forces low slip angles

Brach 970957

- “Analytical Assessment of the Critical Speed Formula”
- Reevaluated Shelton data
- Added some computer modelling
- CONCLUSIONS
 - If 0.1g accelerating, average CSF = 1-2% low
 - If coasting, average CSF = 5% low
 - If 0.2g braking, average CSF = 13.5% low
 - Should measure early in the mark
 - Don’t use CSF when braking on split-co surface

Sledge & Marshek 971147

- “Formulas for estimating vehicle critical speed from yaw marks”
- No testing
- No new data
- No examination of old data
- Derives fundamental formula, then derives several equations that expand on it

Bellion 970955

- “Project Y.A.M. (Yaw Analysis Methodology)”
- 4 different vehicles
- Different steering, braking, throttle inputs
- With and without ABS braking
- Measured to outside edge of tire mark
- Used 15, 20, 30m chords starting at first visible mark

Bellion 970955

- CSF works and “is expected to provide a calculated speed which is less than the actual speed of the vehicle” when used with f-avg.
- Striations show driver inputs: accel. yields closely spaced rearward-pointing striations, braking yields more spread-out forward pointing marks
- CSF on split surfaces using a “resultant” drag factor should yield conservative results
- CSF will often overestimate speed during first turn of a double-steer maneuver and should not be used

Bellion 970955

- Recommended using rear tire mark to be confidently close but usually conservative
- Using raw front tire path (no cg correction) and 15m chord, resulted in under-predicting 94% of the time (103/110)
- Path total-stationed & cg path from cad gave conservative results 22 of 22 cases.
- 41 pages total – all raw data, lots of graphs

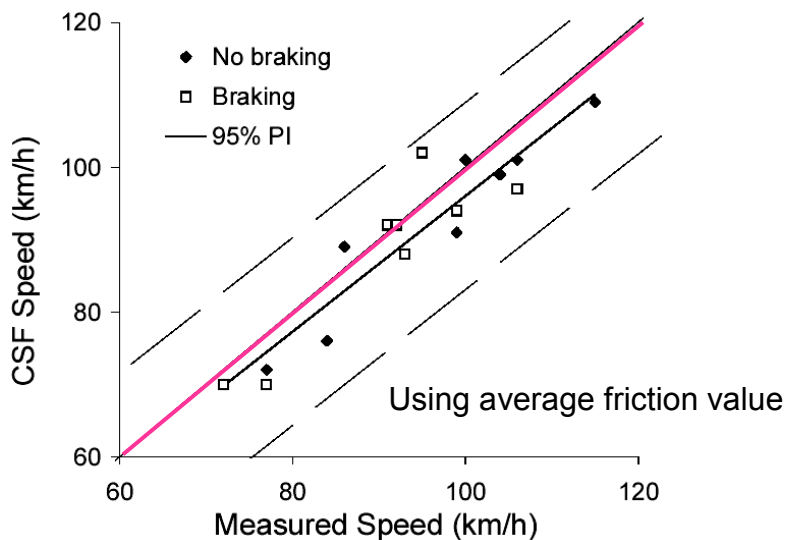
Cliff et al. 2004-01-1187

- “Yaw testing an instrumented vehicle with / without braking”
- 91 Honda Accord
- Yaws with and without light braking
- Measured to outside edge of tiremarks
- Used 10m chords starting at first visible mark
- Subtracted $\frac{1}{2}$ track width to get cg path radius (neglected slip-angle correction)

MacInnis 2004-01-1187

- All tests: 1mph or less lost between steering input and mark
- Results of one test graphically shown:
 - Yaw rate ~ 32deg/sec
 - Lost speed at about 5mph/sec = 0.15g's (right in line with Daily's observations)

MacInnis 2004-01-1187



MacInnis 2004-01-1187

- Under-predicts speed by a little on average, but uncertainty in their measurement gave possible slightly high results for 95th percentile

Table 2. Average error \pm 95% prediction intervals.

Method	μ_{skid}	μ_{peak}
CSF	-3.5 ± 12.9 km/h	9.3 ± 14.8 km/h

-2.2 mph \pm 8.1 mph

More citations

- Bartlett and Wright (2008) *Summary of 56 Recent Critical Speed Yaw Analysis Tests Including ABS and Electronic Stability Control on Pavement, Gravel, and Grass*, ARJ May/June 2008 pp 29-32
- Hague, Lambourn, Turner (1997), *Critical speed studies I: the accuracy of speeds calculated from critical curve marks, and their striations*. Proc. ITAI 3rd Conference, Telford, pp 89-99

More citations

- Hague, Turner, Williams (1997), *Critical speed studies II: the generation of tyre marks by cornering vehicles*. *ibid.* pp 100-102 86
- Lambourn RF (1989), *The calculation of motor car speeds from curved tyre marks*. *J. For. Sci. Soc.*, vol. 29 pp 371-386
- Yamazaki S & Akasaka T (1988), *Buckling behavior in contact area of radial tire structure and skid marks left by tires*. *JSAE Review* vol.9 (3) pp 51-55

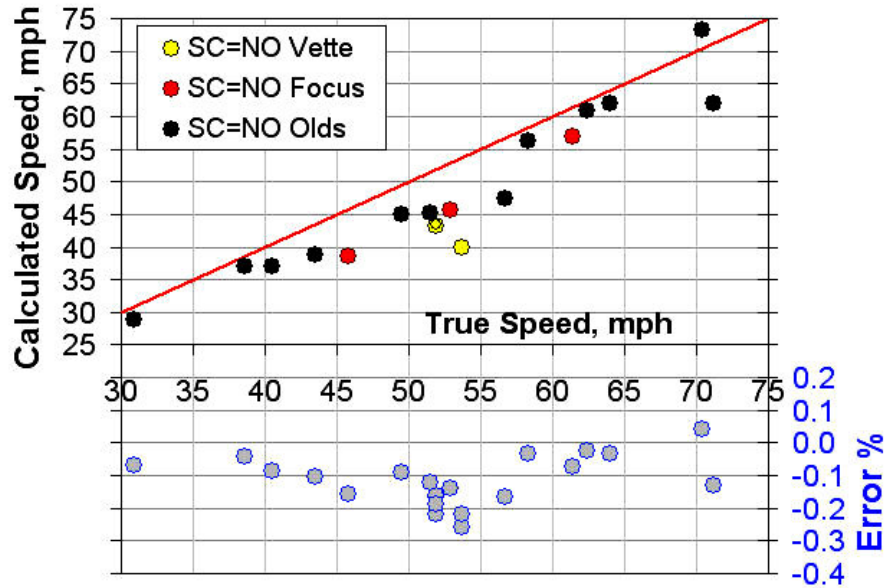
59



Some of Our Tests

- Conducted over a couple years
- Various locations & vehicles
- Speed based on GPS, Vericom, 5th wheels
- 30-ft chords, average deceleration during skid

First 30-foot Chord – Avg.f



Special Cases

- Non ABS Braking
- ABS Braking
- WOT with/without Traction Control
- Stability Control
- Grass, Split-co, Gravel

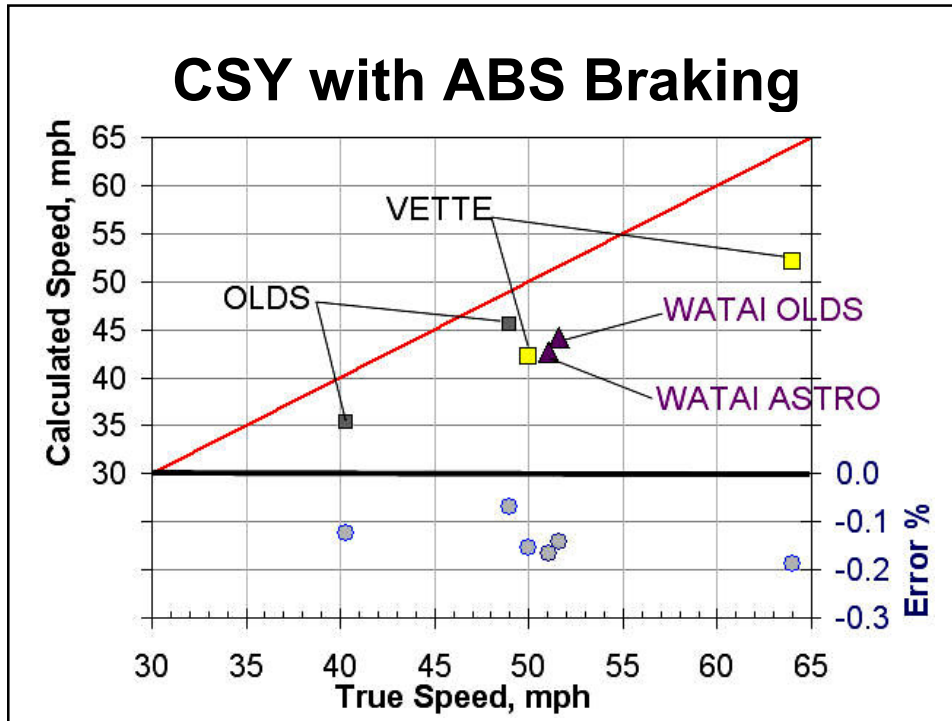
CSY and Non-ABS Braking

- Friction Circle
- Modest braking easily locks wheels
- Careful braking gets inside wheels locked.
- Striation angles will indicate partial braking
- Significant braking effort will lock wheels and end CSY event: Treat as a skid.
- No locked wheels - CSY works

CSY with ABS Braking

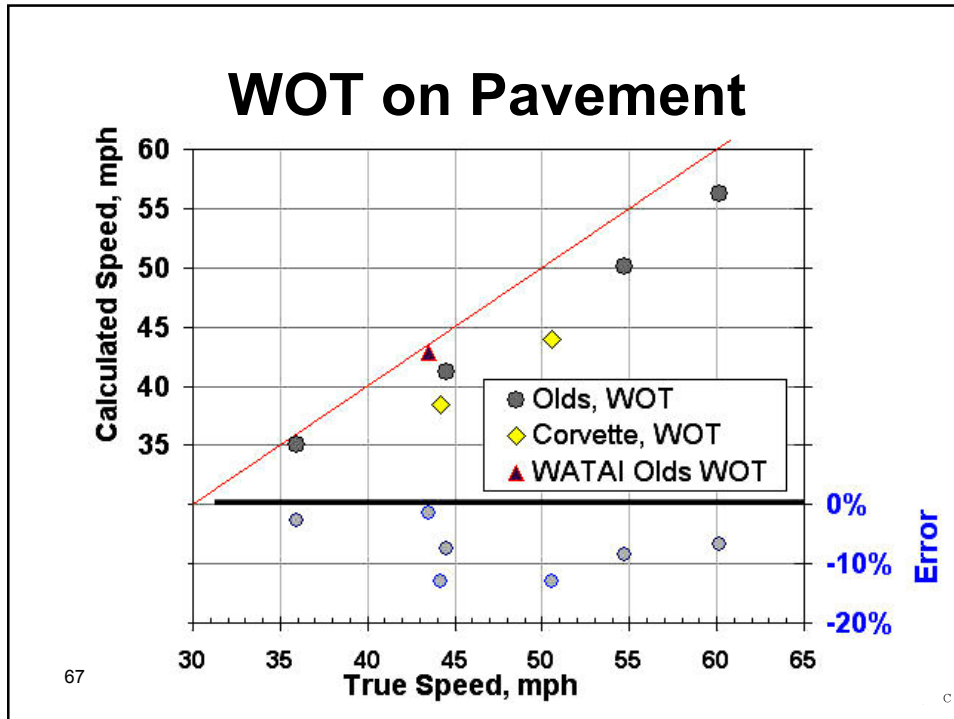
Video 3

- ABS will sense impending skid with laterally saturated tires and release brakes leaving a CSY event: Treat as a CSY
- Tested with Vette (0.4 g decel) and Olds (0.25 g decel)....
- See also Collision 2(2), pg 46, David Dye's article on ABS-affected yaw...Avg. skid friction gives very close results.



CSY and Positive Acceleration No Traction Control

- At speed, possible acceleration is limited
- Friction Circle: 0.3 g long. = still > 90% lat.
- Striation angle and trajectory generally indicate significant acceleration (tilts backward instead of forward).

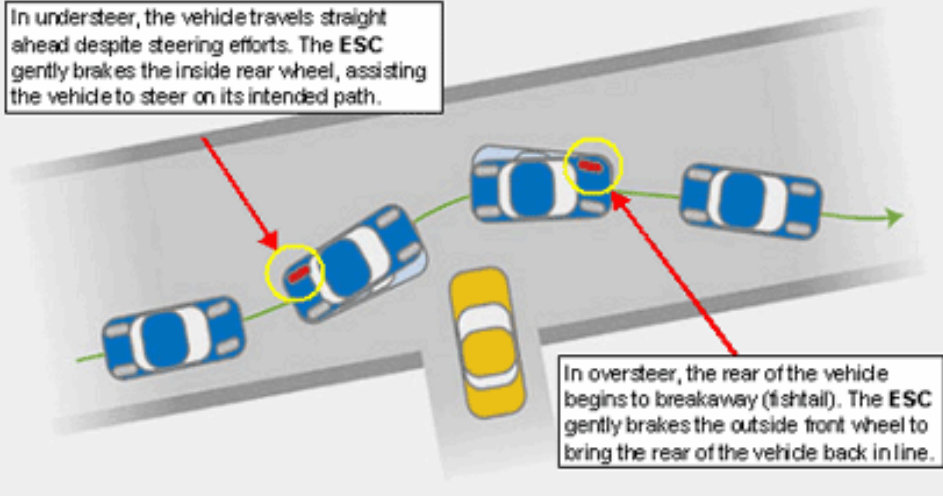


CSY and Positive Acceleration With Traction Control

- Previous results with ABS indicate that the additional tractive force on a laterally saturated tire will invoke traction control and not interfere with CSY model, so we can treat it as a CSY – as long as marks are diverging.

Stability Control

In understeer, the vehicle travels straight ahead despite steering efforts. The ESC gently brakes the inside rear wheel, assisting the vehicle to steer on its intended path.

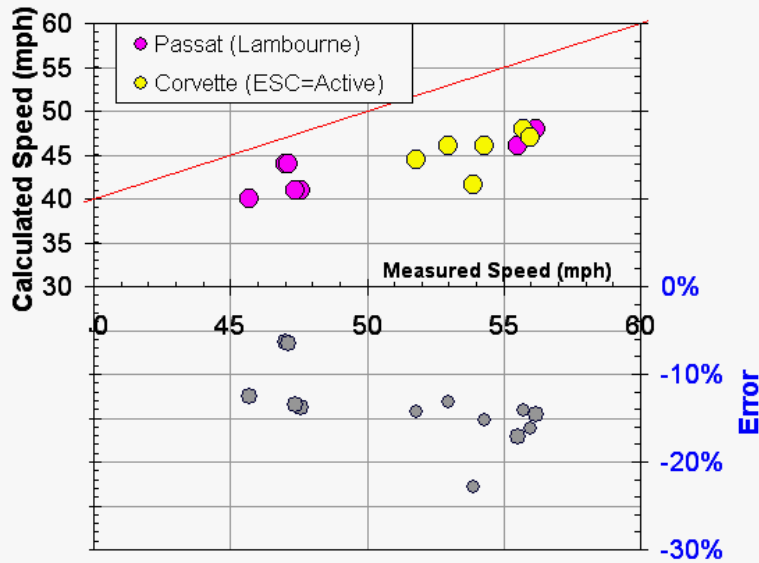


In oversteer, the rear of the vehicle begins to breakaway (fishtail). The ESC gently brakes the outside front wheel to bring the rear of the vehicle back in line.

CSY w/ Stability Control (pavement)

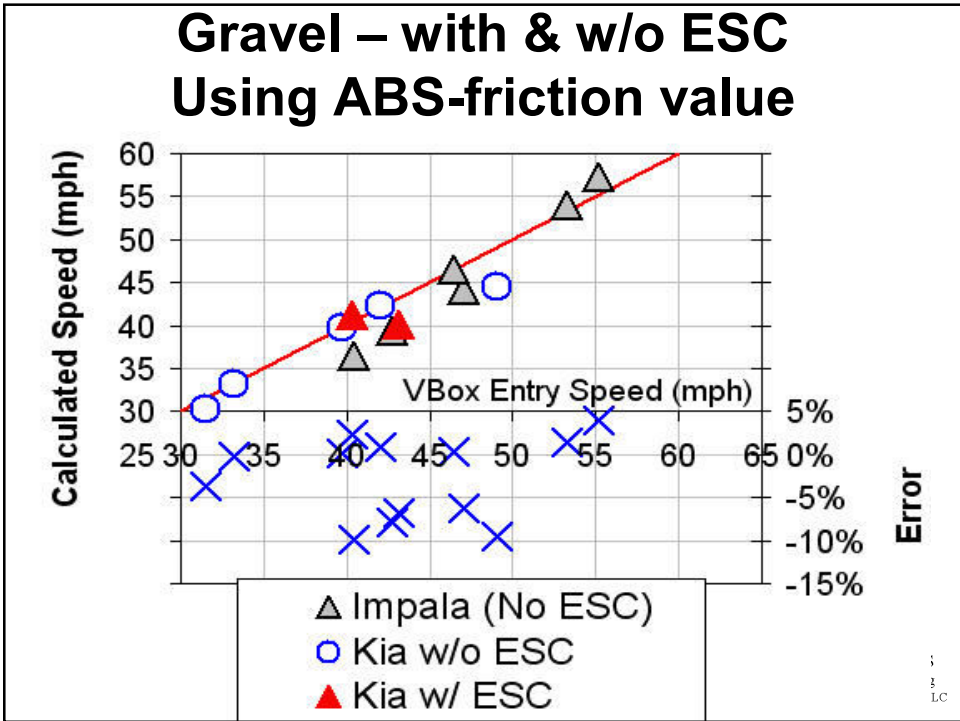
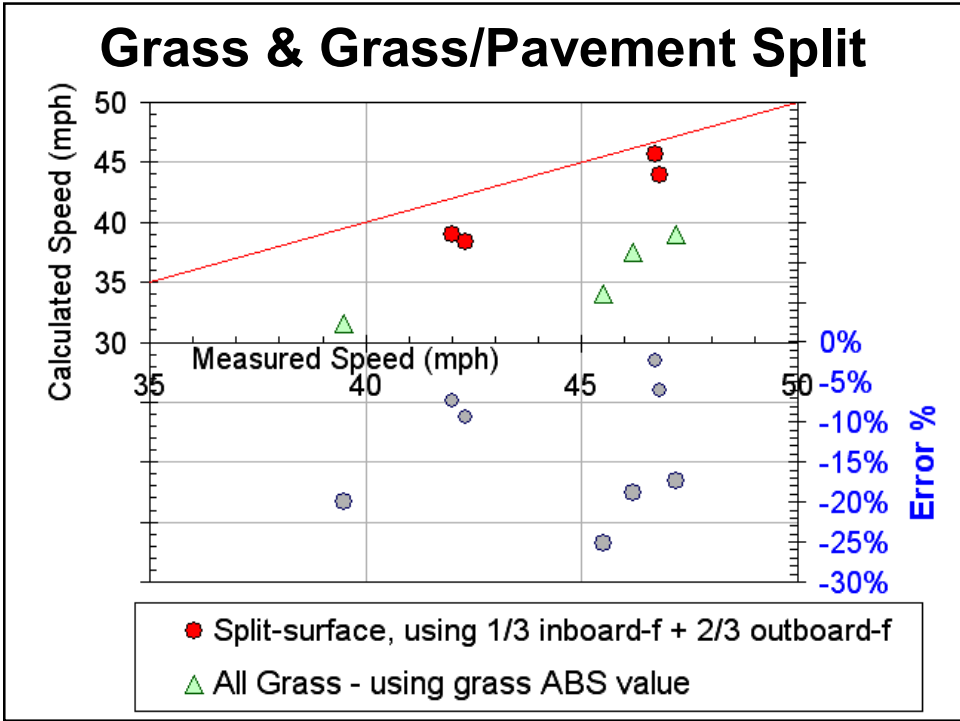
- Friction Circle model suggests the small brake effort will not significantly reduce lateral capability: Treat as a CSY
- Experimental results...

CSY w/ Stability Control (pavement)



CSY w/ Stability Control (pavement)

- Friction Circle model suggests the small brake effort will not significantly reduce lateral capability: Treat as a CSY
- Experimental results: CSY works
- Observation: ESC lets rear end hang out a steady but limited amount.
- No discernable change in marks.



Observations & Recommendations

- 1) When applied properly, Critical Speed Yaw analysis is a reliable and a useful Accident Investigation tool.
- 2) CSY must be based on physical evidence.
- 3) Tire marks should be diverging (vehicle continuing to get further out of shape).

75



Observations & Recommendations

- 4) Not all critical speed events show classic diagonal striations along the tire marks. A lack of such marks does not preclude the use of the model. Absent striated marks, the investigator should rely on vehicle dynamics considerations and an overall perspective of the collision when deciding to apply the model.

76



Observations & Recommendations

- 5) When possible, two successive chords and middle ordinates should be measured. The resulting speed estimates can be compared for consistency.
- 6) Select chord length to achieve a “good” middle ordinate (4 to 10 inches).

77



Observations & Recommendations

- 7) Critical Speed estimates based on average drag factors (average f over an entire locked wheel skid test) yield conservative speed estimates (5% to 20% low).

78



Observations & Recommendations

- 8) ABS-braking during a CSY event does not significantly affect the application of the model. Even with ABS fully activated, vehicles turned into a critical path lose nearly all braking force and resolve the available friction for turning, resulting in a critical speed event.

79



Observations & Recommendations

- 9a) Light non-ABS braking with no locked wheels degrades the lateral friction capability so little that the CSY model works well.
- 9b) Moderate non-ABS braking may result in only one or two locked wheels. This requires delicate brake pedal application and is rarely seen. Evidence of path straightening or locked wheels precludes use of the CSY model.

80



Observations & Recommendations

9c) Firm non-ABS braking will result in vehicle skidding. Once skidding, vehicle's path will become straight. This transition is obvious and precludes use of the CSY model.

81



Observations & Recommendations

10) CSY works on reasonably smooth unpaved surfaces & Split-Coef. events where friction values can be found or estimated.

a) The tire marks must be precluded from being a skid mark. A curved path inconsistent with grade and slope is one good way to accomplish this.

b) Disturbed earth substitutes for diagonal striations along the mark.

82



Observations & Recommendations

- 11) Whenever possible the critical speed estimate should be confirmed with an independent, second speed analysis some place down stream. (DIMS test)
- 12) Determining the radius from the actual mark is preferable to determining the radius using a scaled diagram.

83



Presented by:

Ofc. Wade Bartlett, PE, ACTAR
179 Cross Road
Rochester NH 03867
(603) 332-3267
wade.bartlett@gmail.com
<http://mfes.com>